

TABLES FOR DETERMINATION OF FLOW VARIABLE GRADIENTS BEHIND CURVED SHOCK WAVES

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Department of the Army Project No. 5803-03-001 Ordnance Management Structure Code 5010.11.814 BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

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ABSTRACT

The shock wave relations and the equations of isoenergetic two-dimensional or axisymmetric flows can be combined to yield expressions for various useful quantities behind a shock wave if the curvature is known in addition to the position and slope. Tables are presented here of computations, for $\gamma=1.4$, of seven functions of Mach number and shock wave slope, from which the following are easily computed: 1) streamline curvature, 2) velocity gradient along streamline, 3) angle between streamline and density contour, and 4) angle between streamline and Mach number contour. In addition, the Mach number and angle of inclination of streamline are listed. A Mach number range from 1.1 to 10 is covered by these computations.

A derivation of the shock gradient functions is presented; and several applications of the calculations are given, including a set of tables for determining the slope of the sonic line.

SYMBOLS

Mach number

- angle between streamline and line of constant

ξ

ρ - density

σ - arc length along shock wave

Subscripts

1 - free stream

t - stagnation

I. INTRODUCTION

The problem of determining the gradients of the flow variables behind a shock wave in isoenergetic two-dimensional flow has been considered by several investigators 1,2,3. These gradients are very useful; for instance, the slope of the streamline at shock polar points in the hodograph plane can be obtained from them (the Busemann "hedgehog"). A particularly simple derivation was indicated by Sternberg, using natural coordinates. It is found that the flow variable gradients are proportional to the shock wave curvature. Tables of coefficients for streamline curvature have been computed by Thomas for a limited range of Mach number.

The same gradients can be computed for axisymmetric flow. However, they are now linear combinations of the two shock curvatures, K_g in the meridianal plane and 1/r in the azimuthal plane. (The two-dimensional result in obtained simply by neglecting the 1/r term.) Because of this the general usefulness of these gradient functions is restricted in that one can no longer, as in two-dimensional flow, obtain certain quantities (e.g., slope of sonic line) without having to specify the values of r and K_g . Expressions for the slope of these contours (density, pressure, Mach number) have been given by Wood and Gooderum⁵.

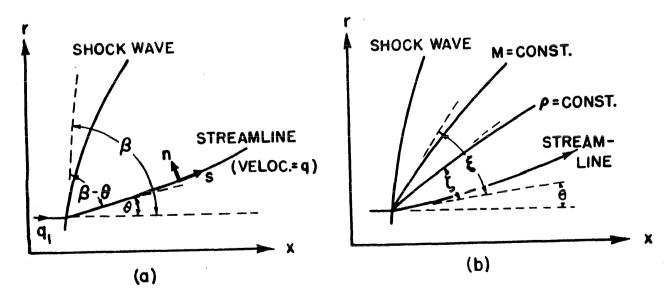


FIGURE 1.1

Aside from the results of Thomas cited above no calculations of the above-mentioned gradient quantities have to the authors' knowledge been published. This report supplies such a set of calculations ("shock gradient functions") which can be used to determine significant flow information from physical measurements. It is felt that these computations can also be helpful in obtaining a clearer general picture of flow behind a curved shock wave. These functions will find further application in the study of flow of a relaxing gas behind shock waves at large Mach numbers.

The particular quantities to be obtained here are (using the notation of Fig. 1.1): 1) $\partial q/\partial s$, the velocity gradient along the streamline; 2) $\partial \Theta/\partial s$ (= K_{ψ}), the curvature of the streamline; 3) $\Theta + \zeta$, the slope angle of the isopycnal, or density contour; and 4) $\Theta + \xi$, the slope angle of the Mach number contour. It is assumed that the gas is ideal, and that the flow is non-viscous and isentropic along streamlines.

The above flow quantities are obtainable from expressions having the following simplified forms:

$$\frac{\partial \Theta}{\partial s} = F_{1}(\gamma, M_{1}, \beta) K_{s} + \epsilon F_{2}(\gamma, M_{1}, \beta) (1/r)$$

$$(1/q_{1})\partial q/\partial s = F_{3}(\gamma, M_{1}, \beta) K_{s} + \epsilon F_{4}(\gamma, M_{1}, \beta) (1/r)$$

$$\tan \zeta = \frac{F_{3}(\gamma, M_{1}, \beta) K_{s} + \epsilon F_{4}(\gamma, M_{1}, \beta) (1/r)}{F_{5}(\gamma, M_{1}, \beta) K_{s} + \epsilon F_{6}(\gamma, M_{1}, \beta) (1/r)}$$
(1.1)

tan \$ =
$$\frac{F_3(\gamma, M_1, \beta) K_s + \epsilon F_4(\gamma, M_1, \beta) (1/r)}{F_7(\gamma, M_1, \beta) K_s + \epsilon F_6(\gamma, M_1, \beta) (1/r)}$$

where ϵ is equal to zero for two-dimensional flow and one for axisymmetric flow. M_1 and q_1 are the Mach number and velocity, respectively, in the free stream; tan β is the slope of the shock wave; γ is the ratio of specific heats. The angle $\theta = \theta$ (M_1, β) is known from the shock wave relations (see, e.g., Ref. 6).

The gradient functions F_1 , F_2 , ..., F_7 are derived in detail in Section II. These functions for the most part are long and complicated expressions in M_1 and β which do not lend themselves to analytical treatment or easy hand computation. It is, however, feasible to calculate them on a high speed computing machine. Thus the functions F_1 , F_2 , ..., F_7 , Θ , and M have been programmed for computation by the EDVAC in the Ballistic Research Laboratories, and they can be obtained quickly for any particular case of supersonic flow by specifying M_1 and γ in the computing machine input.

The simple form of the expressions in Eqs. (1.1) suggests the practicability of a compilation of the coefficients of K_s and 1/r which could be employed as a labor saving means of determining the desired quantities. To this end, the present paper provides a series of tables of shock wave gradient functions versus β for $\gamma = 1.4$, with M_1 as parameter.

The angle ζ is of special interest to the authors, particularly in calculating a flow field (velocity, streamlines, etc.) from given interferometric density data (see Ref. 5, 7, and 8). The information resulting from the measured β and K_s reduces the uncertainty in density near a shock wave which is inherent in the interferometric method.

Sample hand computations have verified that the formulas used in this report yield the same values as formulas given in the references.

II. DERIVATION OF GRADIENT FUNCTIONS

The present derivation follows the one indicated in Ref. 4. In natural coordinates the equations of isoenergetic flow of an ideal non-viscous gas are

$$(1/p) \frac{\partial p}{\partial s} + (1/q) \frac{\partial q}{\partial s} + \frac{\partial 9}{\partial n} + \epsilon (\sin \theta)/r = 0$$
 (a)

$$0 = a\delta/p\delta + pq \, \partial q/\delta = 0$$

$$\partial p/\partial n + \rho q^2 \partial \theta/\partial s = 0$$
 (c) (2.1)

$$h + q^2/2 = constant (d)$$

$$TdS = dh - dp/\rho$$
 (e)

where $\epsilon = 0$ for two-dimensional, = 1 for axisymmetric flow. The coordinates s and n are arc lengths along the streamline and along the orthogonal trajectory of the streamline, respectively; q is the speed of flow, and 0 is the angle between the streamline and the free stream direction (x axis); ρ is the density, and p the pressure. T is the temperature; M is the Mach number $(= q/a, \text{ where } a = (\partial p/\partial \rho)^{1/2} = \text{speed of sound})$.

The two essential equations to be used here are

$$(1 - M2) (1/q) \partial q/\partial s + \partial \theta/\partial n = -\epsilon (\sin \theta)/r$$

$$\partial q/\partial n - q \partial \theta/\partial s = - (T/q) dS/dn,$$
(a)
(2.2)

obtained from Eqs. (2.1) and the definition of the velocity of sound. (The foregoing equations are given in Ref. 9 and 10.) Entropy remains constant along streamlines.

If σ is arc length along the shock wave, it is seen from the relation

$$\partial/\partial \alpha = (\partial/\partial s) (\partial s/\partial \alpha) + (\partial/\partial n) (\partial n/\partial \alpha)$$

that at the shock wave (2.3)

$$\frac{\partial}{\partial \sigma} = (\frac{\partial}{\partial \beta}) \ (\frac{\partial \beta}{\partial \beta}) = K_{\rm S} \frac{\partial}{\partial \beta} = (\frac{\partial}{\partial \rm S}) \cos (\beta - \theta) + (\frac{\partial}{\partial \rm n}) \sin (\beta - \theta)$$

where $\tan \beta$ and K_s are the slope and curvature, respectively, of the shock wave (Fig. 1.1). As the coordinates are defined, curvatures are positive where curves are concave upward and negative where concave downward.

By means of Eq. (2.3), Eqs. (2.2) are replaced by two simultaneous linear algebraic equations for $\partial q/\partial s$ and $\partial \theta/\partial s$. The coefficients of these equations are now converted into the desired form; namely, that of Eqs. (1.1). For this purpose use is made of the following definitions and relations which can be found in many textbooks and reference books (the authors employed Ref. 6 mainly):

$$m = M_1 \sin \beta \tag{2.4}$$

The subscript 1 always refers to free stream conditions.

$$g = \rho_1/\rho = [(\gamma - 1) m^2 + 2] / [(\gamma + 1) m^2]$$
 (2.5)

$$l = p_1/p = (\gamma + 1) / [2\gamma m^2 - (\gamma - 1)]$$
 (2.6)

$$q = A q_1, \qquad (2.7)$$

where

$$A = (\cos^2 \beta + g^2 \sin^2 \beta)^{1/2}$$
 (2.8)

$$\sin (\beta - \theta) = (g \sin \beta)/A$$
, $\cos (\beta - \theta) = (\cos \beta)/A$ (2.9)

$$\sin \theta = \left[(1-g) \sin \beta \cos \beta \right] /A$$
 (2.10)

$$T/T_{1} = g/\ell \tag{2.11}$$

$$M/M_1 = A(\ell/g)^{1/2}$$
 (2.12)

$$S = \left(\frac{q_1}{T_1}\right) \left[\frac{q_1}{\gamma(\gamma - 1)M_1^2}\right] \left[\log_e\left(\frac{1}{\ell}\right) - \gamma \log_e\left(\frac{1}{g}\right)\right]^{(2.13)} + S_1$$

After differentiation for $\partial q/\partial \beta$, $\partial \theta/\partial \beta$, and $\partial S/\partial \beta$ (noting that $\partial S/\partial s=0$), one obtains expression for F_1 , F_2 , F_3 , and F_4 given by the following sequence of formulas:

$$L(m) = g^2 - 1 - 4g/(\gamma m^2 + m^2)$$
 (2.14)

$$K(m) = (1 - m^2 g \ell)^{-1}$$
 (2.15)

$$J(m) = \frac{-\left[g/(\gamma+1)\right] \left[4m^2l - 2(\gamma+1) + 2(\gamma-1)/g\right]}{(\gamma-1) l m^2 L(m)}$$
(2.16)

$$f_1 = g \left[L(1 - J) - K(1 - g) \right]$$
 (2.17)

$$f_2 = -K \left[(L - g + 1)/g + m^2 l L (1 - J) \right]$$
 (2.18)

$$f_z = g K(1 - g)$$
 (2.19)

$$f_{\perp} = K \left[(L - g + 1) + L(1 - J) \right]$$
 (2.20)

$$F_1 = \left[(\cos \beta)/A^{\overline{3}} \right] (f_1 \sin^2 \beta + f_2 \cos^2 \beta) \qquad (2.21)$$

$$\mathbf{F}_{2} = \left[(\cos \beta)/\mathbf{A}^{3} \right] \quad (\mathbf{f}_{3} \sin^{2} \beta \cos \beta) \qquad (2.22)$$

$$F_3 = \left[(\sin \beta)/A^2 \right] (f_4 \cos^2 \beta + g f_3 \sin^2 \beta) (2.23)$$

$$F_{\downarrow\downarrow} = - (g f_3 \sin^3 \beta \cos \beta)/A^2$$

$$\frac{\partial \theta}{\partial s} = F_1 K_s + \epsilon F_2 (1/r)$$
(2.24)

$$(1/q_1) \partial q/\partial s = F_3 K_s + \epsilon F_4 (1/r)$$

The following relations, obtained from the flow equations, are used to find ζ and ξ :

$$\mathbf{p} = \left(\frac{\mathbf{p}_{t}}{\mathbf{p}_{t}^{\gamma}}\right) \mathbf{p}^{\gamma} = \begin{bmatrix} \left(\mathbf{p}_{t}/\mathbf{p}_{t_{1}}\right) & \left(\mathbf{p}_{1}/\mathbf{p}_{t_{1}}\right)^{\gamma} & \mathbf{p}_{1} \\ \left(\mathbf{p}_{t}/\mathbf{p}_{t_{1}}\right)^{\gamma} & \frac{\mathbf{p}_{1}}{\mathbf{p}_{1}/\mathbf{p}_{t_{1}}} \end{bmatrix} \mathbf{p}^{\gamma} \quad (2.25)$$

where the subscript t denotes the stagnation value $(\partial p_t/\partial s = \partial \rho_t/\partial s = 0)$, and

$$p_t/p_{t_1} = \rho_t/\rho_{t_1} = R = i^{1/(\gamma - 1)} g^{-\gamma/(\gamma - 1)}$$
 (See Ref. 6)

$$\rho_{1}/\rho_{t_{1}} = \left[1 + (\gamma - 1) M_{1}^{2}/2\right] -\frac{1}{(\gamma - 1)}$$

$$p_{1}/p_{t_{1}} = \left[1 + (\gamma - 1) M_{1}^{2}/2\right] -\frac{\gamma}{(\gamma - 1)}$$

By means of Eqs. (2.3) and (2.25), $\partial p/\partial s$ and $\partial p/\partial n$ can be expressed in terms of $\partial p/\partial s$, $\partial p/\partial n$, and other quantities already determined in this report. Substituting these expressions into Eqs.(2.1), one then derives expressions for $\partial p/\partial s$ and $\partial p/\partial n$; and putting these into the relation for the angle between the density contour and the streamline, namely,

$$tan \zeta = -(\partial \rho/\partial s)/(\partial \rho/\partial n)$$
 (2.26)

one obtains

A positive value of ζ indicates a counterclockwise rotation from the streamline to the density contour.

The angle 5 between the Mach number contour and the streamline is determined from

$$tan \xi = -(\partial M/\partial s)/(\partial M/\partial n) = -(\partial M^2/\partial s)/(\partial M^2/\partial n), \qquad (2.29)$$

where \$ has the same sign convention as \$. Since

$$M^2 = (\rho q^2)/(\gamma p),$$

 $\partial M^2/\partial n$ can be expressed in terms of the derivatives of ρ and q, which in turn are expressible in terms of previously determined quantities.

Consequently one obtains

$$\tan \xi = \frac{F_3 K_8 + \epsilon F_4 (1/r)}{F_7 K_8 + \epsilon F_6 (1/r)}, \text{ where}$$

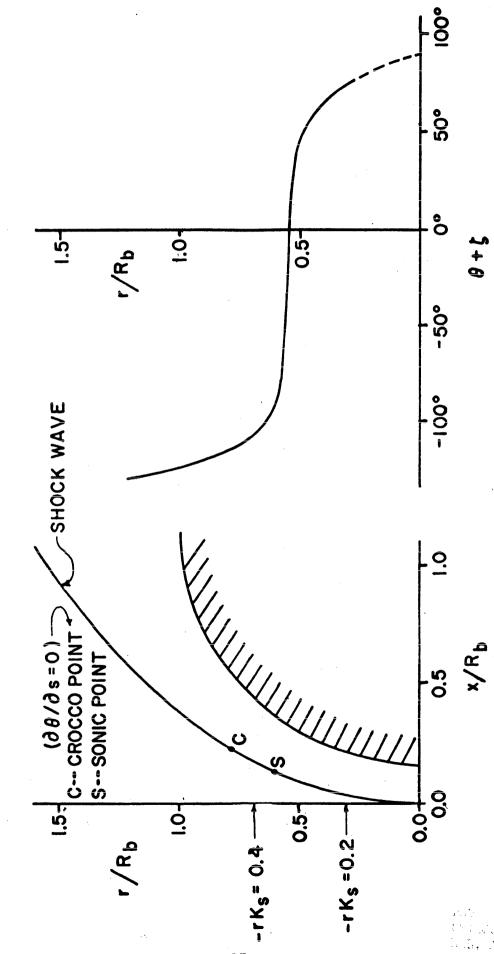
$$F_7 = -\left[\frac{4 \cos \beta}{(\gamma + 1) g^2 m^2 \sin^2 \beta} \begin{cases} A^2 - \frac{2g (m^2 - 1)^2 \sin^2 \beta}{(\gamma + 1) m^2} \\ (\gamma + 1) m^2 \end{cases} \right]$$

$$+ \left\{\frac{(A^3 \gamma l m^2/g) + 2 A \sin^2 \beta}{\sin^2 \beta} F_1 + \left[\frac{A^2 m^2 l \cot \beta}{g^2 \sin^2 \beta} F_3\right] \cdot \left[2 + (\gamma - 1) \frac{A^2 m^2 l}{g \sin^2 \beta} \right]^{-1}$$

III. DISCUSSION OF COMPUTATIONS

Tabulations of the functions F_1 , F_2 , ..., F_7 , 0, and M, calculated for $\gamma = 1.4$, are presented in Section 4 for values of M_1 ranging from 1.1 to 10.0. The functions are listed in order of decreasing β , from $\beta = 90^\circ$ to a value somewhat greater than $\beta = \sin^{-1}(1/M_1)$. The listed values of M_1 and β were chosen so as to permit the determination of the functions to a reasonable degree of accuracy for any M_1 and β by means of graphical interpolation.

Figure 3.1 shows an example of results obtainable from these calculations. The right hand curve shows the variation in slope of the constant density lines along the shock wave of a supersonic sphere in nitrogen (shown at the left) determined from shock wave coordinate measurements and the appropriate shock gradient functions. For reference purposes, the left hand figure shows the sonic point and "Crocco point" (defined below), and indicates the location of typical values of rK_g . It must be noted that the accuracy of results like those in Figure 3.1 is restricted by the limited accuracy of β and K_g



7 = 1.4

M₁= 5.017

SPHERE, R_b= 9/32 IN.

EXAMPLE OF VARIATON OF ANGLE OF CONSTANT DENSITY LINES ALONG A DETACHED SHOCK WAVE. Figure 3.1.

15

determined from experiment. (See Ref. 5 and 8 for discussion on determination of β and $K_{\mbox{\tiny a}}$.)

A point often referred to in two-dimensional flow studies (e.g., Ref. 3) is the "Crocco point", where the streamline curvature is zero. Figure 3.2 shows the geometrical conditions which must exist at the Crocco point in plane and axisymmetric flows for $\gamma = 1.4$. It can be seen that in two-dimensional flow the Crocco point always lies below the sonic point on a shock wave of continuously decreasing slope. The two points approach coincidence with increasing Mach number. On the other hand, in axisymmetric flow the sonic point lies below the Crocco point for all values of M_1 and rK shown. By continuity there would be values of these two parameters for which the sonic point lies above the Crocco point.

The slope of the sonic line is a quantity of considerable interest. It can be otained from the tables by evaluating the β and shock gradient functions corresponding to M = 1 (e.g., by graphical interpolation); it is found to be given by

$$\tan (\Theta + \xi)_{son.} = \frac{B_1 (M_1) rK_s + 1}{B_2 (M_1) rK_s + B_3 (M_1)},$$

where B_1 , B_2 , and B_3 are given in Table 3.1 for $\gamma = 1.4$. It is seen that, formally, $rK_g = -\infty$ corresponds to two-dimensional flow. This particular case is plotted, in slightly different form, in Ref. 11.

Figure 3.3 shows plots of the variation of $\S_{\text{son.}}$, the angle between the streamline and Mach number contour at the sonic point. For two-dimensional flow \S is negative, indicating that the sonic line has a smaller slope than the streamline. For axisymmetric flow the figure indicates that the sonic line can have a greater slope than the streamline. (The streamline curvature must be positive in this case. This can be seen from Eqs. (2.1b) and (2.1c) and the fact that $dq/d\sigma < 0$ and $dM/d\sigma > 0$ along a shock wave with continuously decreasing slope.) If $rK_g = -.2$ at the sonic point, the sonic line and streamline are nearly tangent to each other over a very wide range of Mach number.

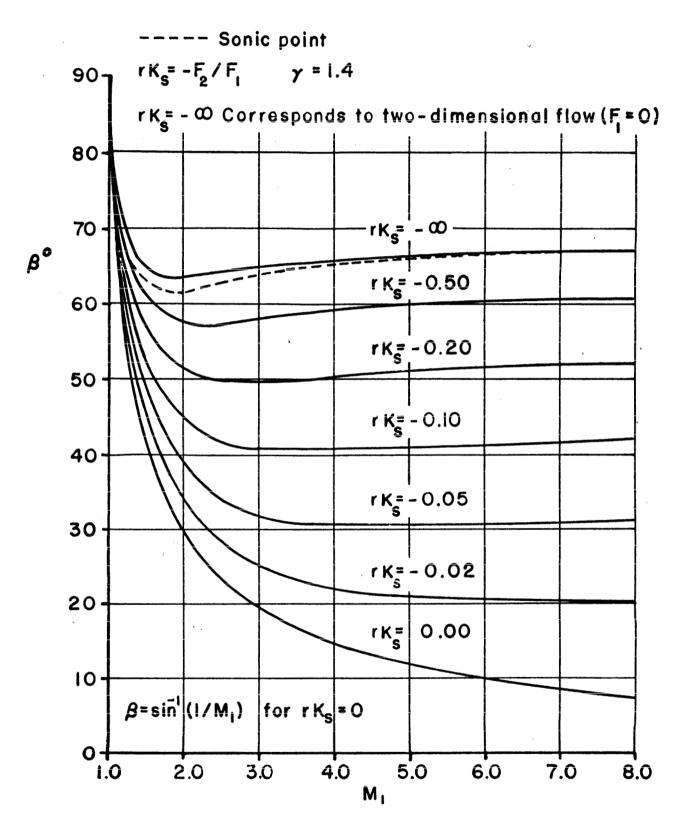


Figure 3.2. Conditions for zero curvature of streamline $(\partial \theta/\partial s=0)$ at shock wave (Crocco Point).

(NOTE: FOR THE $rK_s=0$ CURVE, $\theta+\xi=\beta$)

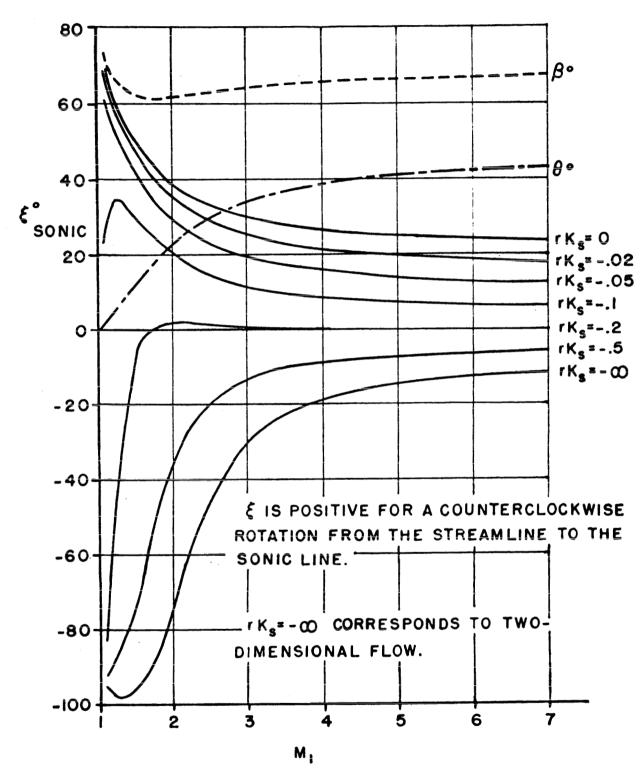


FIGURE 3.3. VARIATION OF ANGLE BETWEEN STREAMLINE AND SONIC LINE AT SHOCK WAVE.

The values of M are also useful in finding the slopes of the characteristics at the shock wave in the supersonic portion of the flow field.

For quick reference, Eqs. (1.1) are restated here.

$$\frac{\partial \Theta}{\partial s} = F_1 K_s + F_2 (1/r)$$

$$(1/q_1)(\partial q/\partial s) = F_3 K_s + F_4 (1/r)$$

$$\tan \zeta = \frac{F_3(rK_s) + F_4}{F_5(rK_s) + F_6}$$

$$\tan \xi = \frac{F_3(rK_s) + F_4}{F_7(rK_s) + F_6}$$

11

TABLE 3.1 Sonic Point Table ($\gamma = 1.4$)

<u> </u>	<u>B</u> 1	<u>B</u> 2	<u>B</u> 3	β ^o	<u> </u>
1.10	10.03	•793	.301	73.26	1.40
1.30	6.15	.175	• 464	65.11	6.31
1.50	4.78	- 0.548	•524	62.26	11.71
1.75	3.71	- 1.40	•546	61.31	17.77
2.00	2.76	- 2.29	•541	61.50	22.70
2.50	1.16	- 3.91	•517	62.66	29.65
3.00	- 0.31	- 5.23	.492	63.77	33.99
3.50	- 1.53	- 6.27	•474	64.62	3 6.82
4.00	- 2.56	- 7.09	.458	65.26	38.74
4.50	- 3.38	- 7.70	•451	65.73	40.06
5.00	- 4.04	- 8,20	• # # #	66.08	41.03
6.00	- 5.04	- 8.92	•435	66.58	42.32
7.00	- 5.70	- 9.38	.428	66,88	43.16
8.00	- 6.19	- 9.72	.424	67.09	43.64
9.00	- 6.50	- 9.92	.420	67.22	44.03
10.00	- 6.76	-10.09	.419	67.35	44.27

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IV - TABLES OF SHOCK GRADIENT FUNCTIONS

 $\gamma = 1.4$

β ^O	ө°	F ₁	10F ₂	F ₃	F ₄	F ₅	10F ₆	F 7	М
		0.0000							

 $M_1 = 1.10$

		_	2)	T	,	0	•	
90	0.000	0.0000	•00000	0.8574	•00000	0.0000	•00000	0.0000	0.912
89	0.169	-0.0543	.00357	0.8502	01495	0.0463	00305	0.0469	0.912
88	0.336	-0.1074	.01421	0.8283	02985	0.0916	01217	0.0928	0.913
87	0.498	-0.1581	.03181	0.7917	04463	0.1351	02728	0.1369	0.915
86	0.655	-0.2054	•05612	0.7399	05923	0.1759	04823	0.1781	0.917
85	0.804	-0.2479	.08683	0.6724	07361	0.2128	07482	0.2156	0.920
84	0.944	-0.2844	.12354	0.5885	08770	0.2450	10681	0.2482	0.923
83	1.072	-0.3136	.16580	0.4871	10147	0.2712	14390	0.2747	0.927
82	1.187	-0.3341	.21309	0.3668	11485	0.2902	18575	0.2940	0.932
81	1.287	-0.3444	• 26484	0.2258	12782	0.3005	23200	0.3046	0.938
80	1.371	-0.3427	• 32045	0.0616	14034	0.3006	28224	0.3048	0.944
78	1.483	-0.2941	• 44071	-0.3513	16390	0.2611	39298	0.2653	0.957
76	1.514	-0.1645	•56876	-0.9167	18532	0.1476	51442	0.1515	0.974
74	1.454	0.0901	• 69956	-1.7246	20447	-0.0838	64288	-0.0805	0.992
72	1.295	0.5642		-2.9781			 77479		1.013
70	1.032	1.5152	• 95145	-5.2381	23582	-1.4444	90674	-1.4430	1.037

 $\gamma = 1.4$ $M_1 = 1.30$

β^{0}	e ^o	$\mathbf{F_{1}}$	F ₂	F ₃	$\mathbf{F_{4}}$	F ₅	F 6	^F 7	M
90	0.000	0.0000	•00000		•00000	0.0000		0.0000.	0.786
89	0.515		•00062		01552		00041	0.0556	0.786
88	1.027		•00248		03095		00164	0.1104	0.787
87	1.532	-0.2335	-		04621		00366	0.1637	0.789
86	2.028	-0.3054	.00973	0.8202	06122		00647	0.2149	0.792
85	2.511	-0.3724	.01501	0.7814	 07590	0.2423	01001	0.2632	0.795
84	2.979	-0.4334	.02127	0.7344	09018	0.2832	01425	0.3079	0.799
83	3.427	-0.4873	•02842	0.6794	10399	0.3201	01914	0.3484	0.804
82	3.855	-0.5335	•03634	0.6168	11727	0.3523	02462	0.3841	0.810
81	4.260	-0.5711	•04491	0.5466	12998	0.3795	03063	0.4146	0.816
80	4.638	-0.5996	.05400	0.4693	14208	0.4011	03711	0.4391	0.823
78	5.309	-0.6269	.07327	0.2938	16429	0.4257	05120	0.4689	0.839
76	5.856	-0.6125	.09317	0.0916	18376	0.4228	06636	0.4699	0.857
74	6.267	-0.5544	•11282	-0.1370	20043	0.3891	08207	0.4386	0.878
72	6.535	-0.4507	.13148	-0.3935	21436	0.3210	09785	0.3715	0.902
70	6.655	-0.2991	.14860	-0.6813	22569	0.2138	11329	0.2637	0.928
65	6.299	0.3298	.18228	-1.5998	24390	-0.2803	14829	-0.2384	1.002
60	5.013	1.5662	.20143	-3.1378	25056	-1.3724	17553	-1.3461	1.089
55	2.836	5.0595	.20685	-7.3220	24903	-4.7334	19342	-4.7248	1.191

 $\gamma = 1.$

$M_1 =$	1.50
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$\beta^{\mathbf{o}}$	e°	F ₁	F ₂	F ₃	\mathbf{F}_{4}	F ₅	F 6	^F 7	M
90	00.00	0.0000	-00000	0.9105	•00000	0.0000	•00000	0.0000	0.701
89	00.86		.00096	-	01587		00052	0.0655	0.702
88	01.72	-0.2159	.00382		03162		00205	0.1301	0.703
87	02.56	-0.3189	.00850		04713	0.1638	00459	0.1929	0.705
86	03.39	-0.4162	.01491		06229	0.2146	00808	0.2531	0.708
85	04.20	-0.5061	.02290	0.8052	07699	0.2621	01248	0.3099	0.712
84	04.99	-0.5872	.03229	0.7604	09114	0.3056	01770	0.3626	0.717
83	05.75	- 0•6583	.04289	0.7088	10466	0.3447	02368	0.4105	0.723
82	06.48	- 0•7184	.05448	0.6507	 11748	0.3787	03033	0.4530	0.729
81	07.17	-0.7669	•06685	0.5868	12955	0.4073	03756	0•4898	0.737
80	07.82	-0.8032	.07977	0.5176	14082		04527	0.5203	0.745
78	09.00	-0.8389	.10644	0.3654	16089		06176	0.5615	0.764
76	10.00	- 0.8262	.13293	0.1982	17762	0.4580	07907	0.5751	0.785
74	10.81	-0.7678	.15799	0.0196	19111		09656	0.5601	0.810
72	11.42	-0.6680	.18070	-	20156		11367	0.5166	0.837
70	11.84	- 0•5309	.20047	-0.3636	20929		12997	0.4442	0.866
65	12.06	-0.0472			21883		16523	0.1332	0.949
60	11.16				21844				1.045
55	09.26				21215				1.152
50	06.44	3.5288	.24037	-3.9945	20270	-3.1431	21319	-3.0970	1.271

 $M_1 = 1.75$

 $\gamma = 1.4$

β ^O	e°	F _l	F ₂	F ₃	$\mathbf{F}_{f l_{f i}}$	F ₅	^F 6	F ₇	М
90	00.00	0.0000	.00000	0.9269	.00000	0.0000	•00000	0.0000	0.628
89	01.28		.00146	0.9221	01615	0.0607	00064	0.0783	0.629
88	02.55	-0.2961	.00580	0.9078	03212	0.1204	- •00255	0.1554	0.630
87	03.80	-0.4359		0.8843	04776	0.1777	00569	0.2301	0.633
86	05.03	-0.5660		0.8520	06289	0.2318	00999	0.3013	0.637
85	06.22	-0.6841	.03430	0.8115	07740	0.2816	01536	0.3681	0.642
84	07.38	-0.7880	.04801	0.7636	09114	0.3264	02168	0.4296	0.648
83	08.50	-0.8761	.06323	0.7089	10403	0.3655	02884	0.4850	0.655
82	09.56		.07957	0.6483	11600	0.3984	03671	0.5339	0.663
81	10.57	-1.0016	.09663	0.5827	12699	0.4247	04514	0.5757	0.672
80	11.52	-1.0385	.11406	0.5130	13697	0 • 4442	05400	0.6103	0.682
78	13.23	-1.0623	.14869	0.3642	15390	0.4626	07252	0.6570	0•705
76	14.68	-1.0255	.18122	0.2077	16693	0.4539	09134	0.6741	0.731
74	15.87	-0.9375	.21012	0.0482	17639	0.4200	10968	0.6629	0.760
72	16.80	-0.8087	.23455	-0.1110	18273	0.3629	12700	0.6253	0.792
70	17.47	-0.6484	.25418	-0.2681	18642	0.2851	- •14290	0.5632	0.826
65	18.12	-0.1583	.28343	-0.6476	18713	0.0123	17524	0.3118	0.922
60	17.48	0.4126	.28988	-1.0210	18036	-0.3636	19666	-0.0709	1.029
55	15.75				17018				1.147
50	13.08				 15898				1.276
45	09.59				14797				1.415
40	05.30	7.0134			13764				1.569
	^ -		•		• • •		• • •	, - , -	1 1 1

β ^O	$\theta_{\mathbf{o}}$	$\mathtt{F}_\mathtt{l}$	F ₂	F ₃	\mathbf{F}_{14}	F ₅	F 6	F ₇	М
90	00.00	0.0000	•00000	0.9375	•00000	0.0000	•00000	0.0000	0.577
8 9	01.66	-0.1930	.00202		01632	0.0652	00076	0.0906	0.578
88	03.32	-0.3807	.00801		03241		00302	0.1797	0.580
87	04.94	-0.5579	.01771		04806		00671	0.2656	0.583
86	06.53	-0.7204	.03073		06304		01172	0.3471	0.588
8 5	08.08	-0.8643	.04657		07721		01794	0.4228	0.594
84	09.57	-0.9869	.06465	0.7546	09040	0.3415	02518	0.4919	0.601
83	10.99	-1.0866	.08436	0.6950	10253	0.3789	03329	0.5534	0.610
82	12.34	-1.1626	.10509	0.6300	11351	0.4089	04208	0.6070	0.619
81	13.61	-1.2149	.12627	0.5606	12332	0.4310	05137	0.6523	0.630
80	14.81	-1.2443	.14736	0.4880	13195	0.4453	06099	0.6891	0.642
78	16.94	-1.2406	.18759	0.3373	14577	0.4511	08060	0.7380	0.669
76	18.73	- 1•1664		0.1847	15538	0.4285	09986	0.7555	0.699
74	20.18	-1.0387	.25268		16136	0.3811	11799	0.7448	0.733
72	21.30	-0.8735			16435	0.3130	13451	0.7092	0.769
70	22.12	- 0•6842			 16498		14915	0.6521	0.808
65	22.97				15966			0.4314	0.917
60	22.41				14919			0.1147	1.037
55	20.73	0.9094			13714			-	1.167
50	18.13	1.5083			12534				1.307
45	14.74	2.2937			11460			-	1.456
40	10.62	3.6349			10524				1.617
35	05.75	7.3335	.18491	-4.6213	09724	-6.9018	17361	-6.8444	1.795

		M ₁ =	2.50		$\gamma = 1.4$					
β ^o	e ^o	Fl	F ₂	F ₃	F_{4}	F ₅	F ₆	F ₇	М	
90	00.00		.00000		-	0.0000	•00000	0.0000	0.513	
89	02.33	- 0.2793	.00320		01652		00096	0.1124	0.514	
88	04.64	-0.5479			03269		00380	0.2224	0.517	
87	06.90	- 0 • 7958			04818		-•00841	0.3276	0.521	
86	09.09	-1.0149	-		06271		01458	0.4260	0.527	
85	11.21	-1.1994			07604		02210	0.5160	0.535	
84	13.22	-1 •3456	.09666		08803		03069	0.5964	0.545	
83	15.12	- 1•4524	-		 09858	0.3914	04008	0.6666	0.556	
82	16.91	- 1.5209		0.5827	10765	0.4131	04999	0.7262	0.568	
81	18.57	-1 •5534	.17807	0.5062	11527	0.4249	06019	0.7753	0.582	
80	20.10	- 1•5536	.20345	0.4286	 12150	0.4273	07044	0.8143	0.597	
78	22.78	-1.4741	•24811	0.2746	13019	0.4064	09042	0.8644	0.631	
76	24.96	-1 •3170	.28300	0.1280	 13462	0.3565	10887	0.8821	0.669	
74	26.68	-1.1134	.30792	-0.0073	 13578	0.2844	12520	0.8735	0.711	
72	27.97	- 0.8867	•32382	-0.1295	13453	0.1959	13917	0.8440	0.756	
70	28.89	-0.6533	.33216	-0.2387	13161	0.0961	15079	0.7979	0.804	
65	29.80	-0.0961	.32971	-0.4619	12031	-0.1816	17053	0.6281	0.934	
60	29.18	0.3904	.30845	-0.6328	10714	-0.4768	17958	0.3966	1.077	
55	27.44	0.8175	.27971	-0.7748	09443	-0.7818	18097	0.1034	1.231	
50	24.85	1.2221	.24923	-0.9102	08307	-1.1076	17697	-0.2682	1.395	
45	21.57	1.6594			07334				1.569	
40	17.68	2.2266			06525				1.754	
35	13.17	3.1621			05875				1.952	
30	07.99	5.4086			05370				2.169	
** ***	1.7 A 1.5					The second second	. .			

β ^O	e ^o	F _l	F ₂	F ₃	$\mathbf{F}_{l_{\!+}}$	F ₅	F 6	^F 7	M
99887654321086777666554433325	00.00 02.85 05.67 08.42 11.07 13.60 15.99 18.22 20.29 22.19 23.93 26.90 29.25 31.06 32.38 33.28 34.07 33.32 31.49 25.62 21.85 17.58 12.77 07.28	-1.7304 -1.7787 -1.7819 -1.7472 -1.5923 -1.3659 -1.1072 -0.8421 -0.5856 -0.0209	.00431 .01687 .03669 .06229 .09189 .12368 .15598 .18737 .21677 .24343 .28699 .31716 .33532 .34371 .34465 .32680 .29544 .26084 .22734 .19659 .14465 .12300	0.9484 0.9237 0.8840 0.8312 0.7678 0.6963 0.6193 0.5393 0.4583 0.3779 0.2240 0.0840 -0.0395 -0.1467 -0.2388 -0.4158 -0.5393 -0.6319 -0.7901 -0.8897 -1.0436 -1.3429	.0000001662032770480406207074610855009469102211081311259117771190211751114181097106308070990605807099060580345803142	0.1530 0.2215 0.2812 0.3302 0.3675 0.3926 0.4058 0.4058 0.4058 0.3560 0.2850 0.1949 0.0924 -0.0175 -0.3039 -0.5874 -0.8604 -1.1276 -1.4046 -1.7277 -2.1835 -3.0261	0011200441009700167102511034530446105502065470757409502111971262613789147071610116532163161566814737136291241911152	0.1298 0.2563 0.3763 0.4872 0.5872 0.6751 0.7504 0.8133 0.8643 0.9043 0.9554 0.9754 0.9754 0.9754 0.9758 0.6285 0.4140 0.1382 -0.2227 -0.7090 -1.4041 -2.5422	0.475 0.476 0.480 0.485 0.493 0.502 0.514 0.527 0.558 0.576 0.616 0.660 0.708 0.760 0.814 0.961 1.123 1.296 1.482 1.681 1.894 2.122 2.367 2.638
		$M_1 = 1$			***************************************		y = 1.4		
β ^O	$\boldsymbol{\Theta}^{\boldsymbol{O}}$	$\mathbf{F}_{\mathbf{l}}$	F ₂	F ₃	F_{14}	F ₅	F ₆	F ₇	М
988765432108642050505050 988765432108642050505050	00.00 03.25 06.46 09.58 12.57 15.40 18.06 20.51 22.77 24.82 26.67 29.80 32.22 34.04 35.36.19 36.85 35.97 34.05 31.39 28.16 24.46 20.35 15.78 10.68 04.77	-0.4242 -0.8240 -1.1782 -1.4715 -1.6952 -1.8477 -1.9326 -1.9572 -1.9313 -1.8647 -1.6480 -1.3709 -1.0764 -0.7892 -0.5220 0.0376 0.4587 0.7836 1.0524 1.3001 1.5642 1.9006 2.4312 3.5499	.02056 .04441 .07471 .10902 .14498 .18049 .21393 .24420 .27061 .31108 .33603 .34818 .35068 .34632 .31858 .28154 .24401 .20925 .17821 .15099 .12736 .10694 .08920	0.9513 0.9231 0.8782 0.8190 0.7489 0.6711 0.5888 0.5047 0.4210 0.3395 0.1875 0.0538 -0.0606 -0.1571 -0.2378 -0.3865 -0.4834 -0.5502 -0.6475 -0.7778 -0.9159 -1.2335	.00000 01667 03279 04784 06142 07328 09141 09774 10244 10567 10855 10789 10493 10057 09545 08170 06870 05737 04780 03986 03338 02820 02420 02130 01942	0.0824 0.1606 0.2310 0.2905 0.3371 0.3700 0.3890 0.3948 0.3713 0.3109 0.2247 0.1223 0.0103 -0.1062 -0.3997 -0.6797 -0.9399 -1.821 -1.4148 -1.6572 -1.9537 -2.4207 -3.4561	15270 15412 14987 14192 13166 12005 10784 09554 08343	-0.7481 -1.5503 -2.9817	0.451 0.452 0.456 0.462 0.471 0.482 0.495 0.5127 0.565 0.565 0.609 0.711 0.767 0.827 0.987 1.163 1.353 1.558 1.779 2.018 2.277 2.558 2.864 3.212

β ^o	θ°	$\mathtt{F}_{\mathtt{l}}$	F ₂	F ₃	\mathbf{F}_{14}	F ₅	^F 6	^F 7	М
9098876584321086777650505050505050505050505050505050505	00.00 03.56 07.07 10.46 13.71 16.76 19.60 22.22 24.60 26.74 28.66 31.87 34.32 36.12 37.39 38.74 37.76 35.77 33.07 29.85 26.20 22.18 17.78 12.94 07.44	0.0000 -0.4790 -0.9270 -1.3176 -1.6327 -1.8634 -2.0098 -2.0787 -2.0811 -2.0295 -1.9368 -1.6726 -1.3598 -1.6721 -0.7423 -0.4704 0.0806 0.4801 0.7782 1.0159 1.2254 1.4366 1.6883 2.0544 2.7353 4.6432	.00607 .02362 .05073 .08471 .12254 .16137 .19885 .23326 .26353 .28912 .32620 .34660 .35404 .35224 .31014 .26979 .23076 .19547 .16444 .13755 .11445 .09475	0.9529 0.9219 0.8727 0.8086 0.7334 0.6509 0.5648 0.4780 0.3929 0.3111 0.1615 0.0332 -0.0743 -0.1631 -0.23663 -0.4469 -0.4984 -0.5623 -0.5623 -0.5623 -0.5645	0167:0327:0327:04763:06084:07215:08145:09805:09424:09996:09616:09996:09616:09128:07211:05960:04891:03998:02665:02188:01545	1 0.0856 7 0.1664 8 0.2378 9 0.3410 9 0.3699 9 0.3837 9 0.3834 9 0.3705 9 0.3467 0 0.2736 0 0.1766 0 0.0655	00133 00523 01140 01943 02880 03903 04965 06026 07057 08036 09786 11227 12363 13226 13852 14616 14574 14018 14018 12047 10850 09615 08396 07230	0.1538 0.3026 0.4420 0.5687 0.6805 0.7766 0.8571 0.9231 0.9759 1.0173 1.0723 1.1001 1.1099 1.1080 1.0981 1.0509 0.9764 0.8682 0.7126 0.4904 0.1759 -0.2698 -0.9183	0.435 0.436 0.441 0.447 0.457 0.469 0.483 0.558 0.559 0.655 0.677 0.659 0.677 0.839 1.096 1.398 1.669 2.412 2.412 2.4728 3.466
•		$M_1 = $	4.50		THE RESERVE OF THE PARTY OF THE)	/ = 1.4		
β	$\Theta^{\mathbf{O}}$	F	⁷ 2	F 3	F_{4}	F ₅	F ₆	F ₇	M
55 50 45 40 35 30 25	00.00 03.80 07.54 11.15 14.58 17.80 20.78 23.50 25.97 28.17 30.14 33.39 35.84 37.63 39.63 40.07 39.01 36.94 31.02 27.41 23.45 19.17 14.51 09.31	-0.5236 -1.0103 -1.4290 -1.7593 -1.9929 -2.1317 -2.1856 -2.1684 -2.0957 -1.9822 -1.6821 -1.3439 -1.0110 -0.7039 -0.4301 0.1122 0.4953 0.7746 0.9914 1.1760 1.3541 1.55555 1.8313 2.3049	.13311 .17396 .21265 .24744 .27734 .30197 .33598 .35269 .35166 .34108 .30281 .26045 .22059 .18510 .15419 .12757 .10485 .08561	0.9206 0.8000 0.8000 0.7208 0.6349 0.5461 0.4576 0.3716 0.2900 0.1428 0.0187 -0.0834 -0.1667 -0.2341 -0.2341 -0.4634 -0.4634 -0.4634 -0.4634 -0.5068	016730327403274047440603507123079980866809150094680964609679096460967908474079230656005352043330279701444	0.0881 0.1707 0.2428 0.3010 0.3432 0.3688 0.3782 0.3731 0.3550 0.3262	01196 02029 02994 04036 05106 06164 07180 08134 09812 11164 12206 12978 13521 14112 13948 13363 12363 10013 08766 07549	7 2040	0.424 0.425 0.437 0.437 0.447 0.460 0.475 0.492 0.533 0.556 0.6661 0.720 0.783 0.849 1.027 1.222 1.669 1.926 2.210 2.526 2.877 3.709

β ^o	$\theta_{\mathbf{O}}$	F _l	F ₂	F ₃	$\mathtt{F}_{\mathtt{1}_{\!\mathtt{1}}}$	F ₅	^F 6	F ₇	М
999 887 888 888 888 888 888 888 888 888	00.00 03.99 07.90 11.68 15.26 18.60 21.68 24.48 27.01 29.26 31.25 34.53 36.98 38.74 40.68 41.04 39.91 37.83 35.09 31.87 28.28 24.37	0.0000 -0.5598 -1.0774 -1.5179 -1.8593 -2.0935 -2.2246 -2.2650 -2.2316 -2.1417 -2.0118 -1.6844 -1.3280 -0.9847 -0.6731 -0.3989 0.1358 0.5064 0.7720 0.9741 1.1415 1.2973 1.4659	.05995 .09901 .14143 .18369 .22314 .25801 .28742 .31113 .34252 .35631 .35749 .35029 .33789 .29675 .25310 .21278 .17723 .14645 .12007	0.9194 0.8641 0.7929 0.7107 0.6222 0.5315 0.4418 0.3555 0.2741 0.1290 0.0084 -0.0898 -0.1689 -0.2323 -0.3410 -0.4033 -0.4386 -0.4579 -0.4684 -0.4756	01674032720472805995070470788008503089370920709342093030899308533080107444061000492703947031400248101950	0.1740 0.2466 0.3041 0.3444 0.3672 0.3732 0.3641 0.3421 0.3092 0.2194 0.1086	001460057101238020940307904133052070625907261081930981311096120681277313255137241347512774117891063409394	0.0000 0.1682 0.3303 0.4809 0.6162 0.7341 0.8341 0.9171 0.9845 1.0385 1.0812 1.1404 1.1758 1.1966 1.2087 1.2155 1.2187 1.2044 1.1647 1.0865 0.9524 0.7396 0.4161	0.415 0.417 0.422 0.429 0.440 0.453 0.469 0.488 0.530 0.554 0.6663 0.724 0.789 0.858 1.041 1.243 1.465 1.709 1.979 2.282 2.622
30 25 20	20.17 15.64 10.67	1.6857 2.0418 2.8144	.07870 .06294	-0.5066 -0.5578	01198 00952	-2.0474 -2.3046 -2.9304	06917 05782	-0.0671 -0.8026	3.006 3.441 3.933
		M ₁ = 0					$\gamma = 1.4$	-200344	
β ^o	$\Theta_{\mathbf{o}}$	F	F ₂	F ₃	$\mathrm{F}_{1_{\!4}}$	F ₅	F ₆	$^{\mathrm{F}}_{7}$	М
99 88 87 86 85 84 83 82	00.00 04.25 08.42 12.43 16.21 19.73 22.94 25.85 28.45	0.0000 -0.6135 -1.1762 -1.6474 -2.0027 -2.2352 -2.3527 -2.3718 -2.3136	.00808 .03115 .06598 .10821 .15329 .19735 .23756	0.9174 0.8581 0.7825 0.6960 0.6040 0.5107	.00000 01675 03267 04703 05934 06937 07709 08266 08634	0.1786 0.2517 0.3079 0.3454 0.3639 0.3648	.00000 00154 00601 01298 02185 03194 04263 05337 06377	0.0000 0.1773 0.3476 0.5049 0.6453 0.7667 0.8689 0.9532 1.0215	0.404 0.406 0.411 0.419 0.431 0.445 0.462 0.482 0.503
81 80 78 76 74 72 70 65 60 55	30.76 32.78 36.08 38.51 40.23 41.39 42.08 42.32 41.11 38.97	-2.1986 -2.0453 -1.6807 -1.3007 -0.9449 -0.6288 -0.3550 0.1676 0.5211 0.7685	.30069 .32285 .35029 .35994 .35743 .34723 .33249 .28782 .24270 .20194	0.3331 0.2523 0.1106 -0.0051 -0.0977 -0.1712 -0.2293 -0.3264 -0.3793 -0.4064	08842 03919 08789 08418 07924 07377 06820 05509 04388 03462	0.3224 0.2840 0.1844 0.0656 -0.0624 -0.1937 -0.3246 -0.6392 -0.9274 -1.1864	07355 08254 09788 10975 11851 12468 12870 13183 12826 12051	1.0763 1.1201 1.1828 1.2239 1.2526 1.2744 1.2926 1.3293 1.3547 1.3609	0.527 0.552 0.607 0.667 0.731 0.799 0.870 1.063 1.274 1.508
50 45 40 35 30 25 20	36.20 32.98 29.42 25.58 21.49 17.13 12.44	1.0973 1.2257 1.3552 1.5110	.13589 .10985 .08778 .06927 .05395	-0.4199 -0.4171 -0.4134 -0.4141 -0.4289	02092 01600 01210 00908 00682	-1.4160 -1.6160 -1.7865 -1.9300 -2.0563 -2.2001 -2.4907	09825 08561 07289 06066 04936	0.5097 -0.0791	1.768 2.059 2.390 2.770 3.210 3.724 4.324

β ^o	e°	$\mathtt{F}_\mathtt{l}$	F ₂	F ₃	$\mathrm{F}_{l_{\!+}}$	F ₅	F ₆	F ₇	М
90 88 88 88 88 88 88 88 88 88 8	00.00 04.43 08.76 12.92 16.83 20.45 23.75 26.73 29.37 31.71 33.75 37.05 39.46 41.16 42.28 43.11 41.84 39.67 36.88 33.65 30.11	0.0000 -0.6499 -1.2429 -1.7339 -2.0971 -2.3269 -2.4338 -2.4377 -2.3624 -2.2306 -2.0623 -1.6744 -1.2804 -0.9178 -0.5997 -0.3269 0.1873 0.5299 0.7663 0.9380 1.0711 1.1837	.00000 .00863 .03319 .07005 .11432 .16106 .20613 .24667 .28105 .30869 .32973 .35446 .36144 .35663 .34462 .32851 .28185 .23598 .19503 .15960 .12927 .10346	0.9159 0.8539 0.7754 0.6862 0.5920 0.4973 0.4055 0.3189 0.2387 0.0993 -0.0131 -0.1022 -0.1722 -0.2271 -0.3173 -0.3647 -0.3871 -0.3941	04685 05892 06862 07595 08111 08437 08650 08467 08063 07551 06999 06445 05158 05158 04072 03181 02456 01871	0.1816 0.2549 0.3102 0.3455 0.3611 0.3586 0.3402 0.3088 0.2669 0.1610 0.0373 -0.0945 -0.2288 -0.3621 -0.6811	12417 11600 10534 09323	0.0000 0.1832 0.3589 0.5206 0.6642 0.7878 0.8913 0.9763 1.0453 1.1007 1.1453 1.2107 1.2559 1.2902 1.3188 1.3446 1.4041 1.4565 1.4788 1.5065 1.4788 1.3926	0.399 0.404 0.413 0.425 0.440 0.458 0.458 0.525 0.551 0.669 0.736 0.879 1.537 1.808 2.114 2.465
35 30 25 20	26.31 22.28 18.04 13.52	1.2914 1.4128 1.5811 1.8815	.08163 .06335 .04827 .03612	-0.3724 -0.3629 -0.3608	01403 01034 00750 00538 00388	-1.9869 -2.0992 -2.1977 -2.3494	06764 05536 04408 03416	1.2218 0.9265 0.4419 -0.3542	2.875 3.360 3.941 4.641
_		$M_{1} = $	8.00			•	y = 1.4		
β ^O	e°	F _l	F ₂	F ₃	$\mathtt{F}_{\underline{l}_{4}}$	F ₅	^F 6	F ₇	М
998876543210864205050505050 9098876543220	00.00 04.55 09.00 13.26 17.26 20.95 24.30 27.32 29.99 32.34 34.39 37.70 40.10 41.77 42.87 43.62 42.32 40.12 37.32 34.09 30.56 22.80 18.63 14.22	-2.0717 -1.6685 -1.2655 -0.8989 -0.5798 -0.3080 0.2003 0.5357 0.7648 0.9292 1.0542 1.1569 1.2511 1.3516 1.4831	.16636 .21206 .25274 .28682 .31385 .33407 .35693 .36210 .35579 .34259 .32562 .27774 .23143 .19041 .15506 .12487 .09922 .07755 .05942	0.9148 0.8510 0.7704 0.6794 0.5837 0.4881 0.3959 0.3094 0.2296 0.0919 -0.1050 -0.1727 -0.2255 -0.3113 -0.3552 -0.3747 -0.3789 -0.3789 -0.3467 -0.3467 -0.313 -0.3198	.00000 01677 03260 04672 05863 06810 07516 08004 08303 08448 08469 08253 07308 06754 06202 04934 0303 02299 01733 02299 01733 00927 00655 00453 00311	0.3454 0.3590 0.3540 0.3540 0.3331 0.2992 0.2549 0.1449 0.0179 -0.1165 -0.2527 -0.3876 -0.7099 -1.0047 -1.5080 -1.5080 -1.7157 -1.8925 -2.0364 -2.1464 -2.275	02281033140439305463054630648507433082920973110814115931212012443126051214511300102160899207705064180518704057	0.0000 0.1873 0.3666 0.5313 0.6771 0.8020 0.9064 0.9919 1.0613 1.172 1.1624 1.2298 1.2781 1.3163 1.3497 1.3810 1.4565 1.5279 1.5882 1.6272 1.6322 1.6322 1.6322 1.6322 1.6322 1.5867 1.5867 1.5867 1.529	0.393 0.395 0.400 0.409 0.422 0.437 0.455 0.455 0.455 0.550 0.672 0.550 0.672 0.885 1.836 2.551 2.551 2.551 4.107 4.896

β ^o	e°	Fl	F ₂	F ₃	$\mathtt{F}_{\underbrace{1}_{+}}$	F ₅	F ₆	F ₇	М
99 88 88 88 88 88 88 88 88 88	00.63 09.16 13.49 17.56 21.30 24.69 27.73 30.42 32.79 34.81 54 42.28 43.99 43.	-1 • 2546 -0 • 8853 -0 • 5657 -0 • 2947 0 • 2093	.03566 .07491 .12155 .17013 .21624 .25696 .29080 .31737 .336949 .36240 .35505 .34106 .327481 .22822 .18717 .15189 .12181 .09627 .07469 .04186	0.9139 0.8489 0.7669 0.6746 0.5779 0.4816 0.3892 0.3027 0.2233 0.0868 -0.0218 -0.1730 -0.2243 -0.3685 -0.3661 -0.3685 -0.3661 -0.3473 -0.3473 -0.3295 -0.32932	.00000 01677 03258 04663 05842 06772 07460 07928 083342 08342 08364 07667 07140 06585 06036 06781 06781 06583 06036 000856 000856 000856 000856 000856	0.1851 0.2585 0.3124 0.3452 0.3573 0.3576 0.3279 0.2922 0.2463 0.1334 0.0041 -0.1321 -0.2697 -0.4058 -0.7304 -1.0275 -1.2962 -1.5365 -1.7479 -1.9289 -2.0770 -2.1893 -2.2654	001650064301381023080230803347044280549606512074500829809709107641151512018123211244211955120920876307469087630494403814	0.000 0.1902 0.3721 0.5389 0.6861 0.8120 0.9170 1.0029 1.0726 1.1289 1.1745 1.2435 1.2940 1.3351 1.3720 1.4072 1.4942 1.5794 1.6561 1.7150 1.7445 1.7445 1.7445 1.7445 1.7445 1.7450 1.7445	0.397 0.397 0.419 0.435 0.455 0.455 0.455 0.455 0.674 0.674 0.888 0.674 0.888 0.674 0.888 0.655 0.674 0.888 0.655 0.674 0.655 0.667
	······································	M ₁ = 1					v = 1.4		
β ^o	e°	Fl	F ₂	F ₃	$\mathtt{F}_{)_{\!$	F ₅	F ₆	F ₇	М
90 90 90 90 90 90 90 90 90 90	00.00 04.70 09.28 13.67 17.78 21.55 24.98 28.03 30.74 33.11 35.17 38.47 40.86 42.50 43.57 44.28 40.68 42.89 40.68 37.84 31.09 27.34 23.41 19.32 15.05	-0.7076 -1.3479 -1.8686 -2.2419 -2.4650 -2.5531 -2.5318 -2.4295 -2.2720 -2.0810 -1.6596 -1.2464 -0.8753 -0.5555 -0.2850 0.2158 0.5424 0.7630 0.9188 1.0345 1.1259 1.2047 1.2821 1.3738	.00951 .03643 .07642 .12377 .17288 .21927 .269366 .319935 .35935 .35935 .35935 .35935 .35936 .32190 .272589 .18488 .321966 .22589 .184960 .1194166 .07266 .0395	0.9560 0.9133 0.8473 0.7642 0.6710 0.5736 0.4769 0.3843 0.2979 0.2188 0.0243 -0.1081 -0.1731 -0.2234 -0.3041 -0.3600 -0.3611 -0.3524 -0.33174 -0.3957 -0.2747	0167703256032560465605826067450787308140082540825007996075510702006465059180467303639027990212001576	0.1861 0.2595 0.3131 0.3451 0.3560 0.3481 0.2871 0.2871 0.2899 0.1249 -0.060 -0.1435 -0.2821 -0.4454 -1.3151 -1.5580 -1.7724 -2.1099 -2.2262 -2.3029	001670065001394023270337104453055200653107462083010969110726114571194312231123231161810942098370859807300060060477003639	0.000 0.1923 0.3761 0.5444 0.6928 0.8194 0.9248 1.0110 1.0808 1.1374 1.1834 1.2535 1.3057 1.3490 1.3885 1.4266 1.5222 1.6176 1.7066 1.7804 1.8381 1.7693 1.8381 1.7693 1.8381 1.7693 1.8381 1.7693 1.8381 1.7693 1.8381 1.7693 1.8381 1.7693 1.8381 1.8381 1.7693 1.8381 1.8	0.388 0.390 0.395 0.405 0.417 0.452 0.474 0.452 0.474 0.5550 0.674 0.893 1.092 1.871 2.581 2.581 3.620 4.337 5.267

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